The Effect of the Substrates Two-Dimensional Temperature Distribution on the TEC Performance

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Abstract

Temperature distribution on a thermoelectric cooler (TEC) cold surface is of high practical value, as the sizes of the cooled object may not coincide with the dimensions of the TEC cold side and it is necessary to make the object temperature closer to the average cold substrate temperature. It is also very important to take into account the temperature distribution on the intermediate substrates of multistage TECs both in mathematical simulation and design modeling.

The approach to finding the approximate two-dimensional temperature distribution for the case of a heat source located on the surface has been developed in papers [1,2]. In this paper this method is analytically verified and applied to calculations of the temperature 2D-profiles of the TEC substrates. Application of the above-mentioned method for performance improvement of TEC systems is discussed.

Two-Dimensional Temperature Distribution on the Cold Substrate of a Single-Stage TEC

Consider a problem of the temperature distribution on the cold substrate surface of a single-stage TEC. Assume it consists of N pellets. There is a heat source localized on the TEC cold substrate.

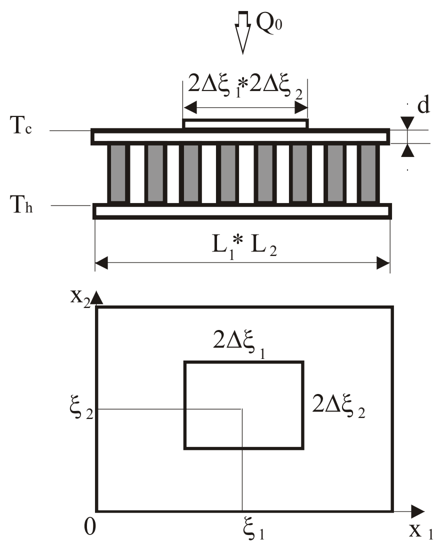


Figure 1 Schematic view of the heat source on a single-stage TEC cold substrate surface

Suppose the TEC substrate is an L_1 x L_2 rectangular, and the heat source is a $2\Delta\xi_1$ x $2\Delta\xi_2$ one. The heat source centre coordinates are ξ_1, ξ_2. The heat source load to be pumped by the TEC equals Q_0. The hot surface temperature is a fixed value T_h, and the cold surface temperature is a two-dimensional function $T_c(x_1,x_2)$. Hereinafter, not to take into account discreteness of pellets on the substrate surface, we do not restrict each pellet cooled (heated) area to the pellet cross-section s_0, but regard it as the full substrate area per a pellet – L_1L_2/N. That is, we assume a quasi-continuous pellets distribution on the substrate surface. Within this approach the calculated temperature two-dimensional field differs from the real one by the absence of a slight periodicity (its period equals the distance between pellets). Then the pellets 2D-distribution density is equal to $\frac{N}{L_1L_2}$. Ignoring temperature dependences of thermoelectric parameters, we consider the Seebeck coefficient α, thermal conductivity κ and electrical resistivity ρ to be constant values. When the pellet is exposed to the electrical current I, the heat flux q_{pellet} [3] is pumped to the pellet cold end:

$$q_{pellet} = -\alpha I T_c + \frac{1}{2} I^2 R + k(T_h - T_c), \qquad (1)$$

where the first term on the right side of equation (1) expresses the Peltier heat extracted by the pellet from the substrate, the second term is the part of the Joule heat, arriving at the substrate from the pellet, and the third term describes the heat flux coming from the hot substrate by the pellet thermal conductance. Here α – the Seebeck coefficient, $R = \rho \frac{l}{s_0}$ – pellet electrical resistance, $k = \kappa \frac{s_0}{l}$ – pellet thermal conductance, l – pellet length. Let d denote the substrate thickness and λ stand for the substrate thermal conductivity. Then the heat conductance equation can be written as follows:

$$\lambda d\left(\frac{\partial^2 T_c}{\partial x_1^2}\right) + \lambda d\left(\frac{\partial^2 T_c}{\partial x_2^2}\right) - \frac{N(\alpha I + k)T_c}{L_1L_2} + \frac{N\left(\frac{1}{2} I^2 R + kT_h\right)}{L_1L_2} + \frac{Q_0 1\{u\}}{4\Delta\xi_1\Delta\xi_2} = 0, \qquad (2)$$

where we write the symbol $1\{u\}$ for the function equal 1 within the area of the heat source Q_0 and 0 within the rest of the surface. Suppose the heat is only removed from the cold substrate by the pellets and there are no lateral heat fluxes:

$$\left.\frac{\partial T_c}{\partial x_i}\right|_{x_i=0,\, x_i=L_i} = 0, \ \ i = 1, 2. \qquad (3)$$

If turning the current I into the reduced current $j = \frac{Il}{s_0}$ and denoting the pellets filling coefficient K_f:

$$K_f = \frac{Ns_0}{L_1L_2}, \qquad (4)$$

we define:

$$b^2 = \frac{(\alpha j + \kappa)K_f}{\lambda l d}, \qquad (5)$$

$$A = \frac{K_f\left(\frac{1}{2} j^2 \rho + \kappa T_h\right)}{\lambda l d}, \qquad (6)$$

$$C = \frac{Q_0}{4\Delta\xi_1\Delta\xi_2\lambda d}. \qquad (7)$$

Making in (2) the substitution of variables

$$T_c = \theta + \frac{A}{b^2}, \tag{8}$$

we obtain the following equation:

$$\frac{\partial^2\theta}{\partial x_1^2} + \frac{\partial^2\theta}{\partial x_2^2} - b_1^2\theta + C_1 1\{u\} = 0 \tag{9}$$

with boundary conditions:

$$\left.\frac{\partial\theta}{\partial \overline{x}_i}\right|_{\overline{x}_i=0,\overline{x}_i=1} = 0, \qquad i = 1,2\,. \tag{10}$$

The approximate solution of this problem is known and given in papers [1,2]:

$$\theta = \frac{C_1}{b_1^2}\phi_1\phi_2\,. \tag{11}$$

In dimensionless coordinates

$$\overline{\mathrm{x}}_{\mathrm{i}} = \frac{\mathrm{x_i}}{\mathrm{L_i}}, \overline{\xi}_{\mathrm{i}} = \frac{\xi_{\mathrm{i}}}{\mathrm{L_i}}, \Delta\overline{\xi}_{\mathrm{i}} = \frac{\Delta\xi_{\mathrm{i}}}{\mathrm{L_i}}, \qquad \mathrm{i} = 1,2 \tag{12}$$

the functions ϕ_i have the view:

$$\phi_i = \begin{cases} K_i ch(p_i\overline{x}_i), \ x_i \in [0;\overline{\xi}_i - \Delta\overline{\xi}_i] & K_i = \dfrac{2sh(p_i\Delta\overline{\xi}_i)ch(p_i(1-\overline{\xi}_i))}{shp_i} \\ K_i ch(p_i\overline{x}_i) - ch(p_i(\overline{x}_i - \overline{\xi}_i + \Delta\overline{\xi}_i)) + 1, \ x_i \in [\overline{\xi}_i - \Delta\overline{\xi}_i, \overline{\xi}_i + \Delta\overline{\xi}_i] & \\ K_i ch(p_i\overline{x}_i) - ch(p_i(\overline{x}_i - \overline{\xi}_i + \Delta\overline{\xi}_i)) + ch(p_i(\overline{x}_i - \overline{\xi}_i - \Delta\overline{\xi}_i)), \ x_i \in [\overline{\xi}_i + \Delta\overline{\xi}_i, 1] & \end{cases}$$

i = 1,2, (13)

where

$$p_i = \frac{L_i}{L_{i+(-1)^{i-1}}}\sqrt{B_{i+(-1)^{i-1}}\left[1{,}5 - \left(\frac{sh\left(2\left(\sqrt{B_{i+(-1)^{i-1}}}\right)\right)}{2\left(\sqrt{B_{i+(-1)^{i-1}}}\right)} + 1\right)^{-1}\right]}, \ i = 1,\ 2, \tag{14}$$

$$B_i = b^2 L_i^2, \quad \mathrm{i} = 1{,}2. \tag{15}$$

Therefore, the temperature distribution for the case in Figure 1 is yielded by the expression:

$$T_c = \frac{Q_0 l}{S_q K_f(\alpha j + \kappa)}\phi_1\phi_2 + \frac{\frac{1}{2}j^2\rho + \kappa T_h}{\alpha j + \kappa}, \tag{16}$$

where S_q – the area covered by the heat source.

Two-Dimensional Temperature Distribution on the Intermediate Substrate of a Two-Stage TEC

Consider now a two-stage TEC operation. Let the first (upper) stage (cascade) cover the area $2\Delta\xi_1 x 2\Delta\xi_2$ and consist of N_1 pellets. The heat load delivered onto the first stage is q_0. Let the second stage cover the area L_1 x L_2 and have N_2 pellets. Not to be concerned about the type of electrical connection we suppose power is supplied to the cascades independently, so that we can be free choosing the pellets geometry of each cascade. Thus we denote the pellets cross-section s_i and the pellets height l_i, where indices i=1,2 correspond to the stage number. Thermoelectric parameters values are also taken different per stage and further are distinguished by its stage index. The current densities for two stages can also differ and we denote them j_i, i = 1,2. Our objective is obtaining temperature distribution $T_c(x,y)$ on the intermediate substrate between the first and the second cascade.

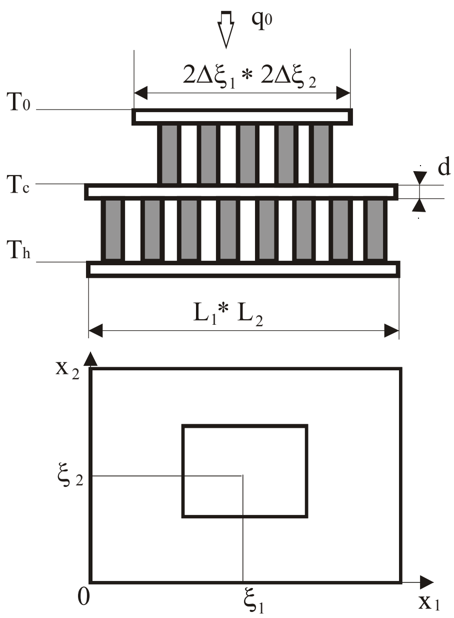


Figure 2 Schematic view of geometry and temperatures on the cascades of a two-stage TEC.

Unfortunately, solution [1] is obtained for the uniform heat load on the cooled substrate, that is why we deal with the approximate solution, assuming the heat flux from the first stage to the second one is evenly spread over their contact area. We also suggest the heat load on the upper stage be uniform as well and the pellets of both cascades be arranged quasi-continuously, as we supposed in the single-stage problem. Let us write Q_0 for the heat flux from the first stage. It can be expressed as follows:

$$Q_0 = \frac{N_1 s_1}{l_1}\left[\alpha_1 j_1 \overline{T}_c + \frac{1}{2} j_1^2 \rho_1 - \kappa_1\left(\overline{T}_c - T_0\right)\right], \tag{17}$$

Here $\overline{T}_c$ is the average temperature of the contact area between two stages:

$$\overline{T}_c = \frac{1}{4\Delta\xi_1\Delta\xi_2}\int_{\xi_1-\Delta\xi_1}^{\xi_1+\Delta\xi_1}\int_{\xi_2-\Delta\xi_2}^{\xi_2+\Delta\xi_2} T_c\, d\xi_1 d\xi_2 \,. \tag{18}$$

Analogous heat rate equations for the cold substrate of the first stage allows eliminating the temperature T_0 from Eq. (17):

$$Q_0 = \frac{N_1 s_1}{l_1}\left[\frac{\alpha_1^2 j_1^2 \overline{T}_c + \frac{1}{2} j_1^2 \rho_1\left(\alpha_1 j_1 + 2\kappa_1\right) + \kappa_1 q_0 \frac{l_1}{N_1 s_1}}{\alpha_1 j_1 + \kappa_1}\right]. \tag{19}$$

Therefore, we come to the equation similar to Eq. (2). Its solution is given by the following expression:

$$T_c = \frac{c}{\beta^2}\phi_1\phi_2 + \frac{a}{\beta^2}, \tag{20}$$

where the functions ϕ_1 and ϕ_2 are determined by (12) – (15), and the other terms are given below:

$$c = \frac{K_{f1}}{\lambda d l_1}\left[\frac{\alpha_1^2 j_1^2 \overline{T}_c + \frac{1}{2} j_1^2 \rho_1 (\alpha_1 j_1 + 2\kappa_1) + \kappa_1 q_0 \frac{l_1}{N_1 s_1}}{\alpha_1 j_1 + \kappa_1}\right], \tag{21}$$

$$a = \frac{K_{f2}\left(\frac{1}{2} j_2^2 \rho_2 + \kappa_2 T_h\right)}{\lambda d l_2}, \tag{22}$$

$$\beta^2 = \frac{K_{f2}(\alpha_2 j_2 + \kappa_2)}{\lambda d l_2}. \tag{23}$$

As a result of these transformations the value $\overline{T}_c$ remains unknown. It can be found by a multi-iteration procedure. For a zero approximation we take $\overline{T}_c$ as a solution of linear equations, describing heat balance on the TEC substrate without allowing for heat losses and temperature distribution (that is, in the one-dimensional approach):

$$\overline{T}_c = \frac{4\Delta\xi_1 \Delta\xi_2}{L_1 L_2}\frac{c}{\beta^2} + \frac{a}{\beta^2}. \tag{24}$$

The solution is

$$\overline{T}_c = \frac{\frac{N_2 s_2}{l_2}\left(\frac{1}{2} j_2^2 \rho_2 + \kappa_2 T_h\right)(\alpha_1 j_1 + \kappa_1) - \frac{N_1 s_1}{l_1}\frac{1}{2}\alpha_1 j_1^3 \rho_1 + q_0 \kappa_1}{\frac{N_2 s_2}{l_2}(\alpha_2 j_2 + \kappa_2)(\alpha_1 j_1 + \kappa_1) - \frac{N_1 s_1}{l_1}\alpha_1^2 J_1^2} \tag{25}$$

After the temperature distribution is found in the first iteration, we can carry out the integration over the thermal contact area and calculate $\overline{T}_c$. As the solution of the heat conduction equation is expressed in the analytical form, the correspondent integrals are easily calculated. If denoting

$$\varphi_i = \frac{\int_{\bar{\xi}_i - \Delta\bar{\xi}_i}^{\bar{\xi}_i + \Delta\bar{\xi}_i} \phi_i(x)dx}{2\Delta\bar{\xi}_i} = \frac{\left[2K_i\left(chp_i\bar{\xi}_i shp_i\Delta\bar{\xi}_i\right) - sh2p_i\Delta\bar{\xi}_i\right]}{2p_i\Delta\bar{\xi}_i} + 1,\ i = 1,2, \tag{26}$$

the expression for $\overline{T}_c$ can be written as follows:

$$\overline{T}_c = \frac{c}{\beta^2}\varphi_1\varphi_2 + \frac{a}{\beta^2}. \tag{27}$$

With the help of Eq. (27) we can find the value $\overline{T}_c$, calculate a new value of c_1 from Eq. (21) and, with it, find a new $\overline{T}_c$ and etc. The procedure described above converges quickly and only a few iterations are required.

Due to the heat losses the average temperature of the thermal contact area $\overline{T}_c$ is different from the average temperature of the whole intermediate substrate $\overline{T}_{cl}$:

$$\overline{T}_{cl} = \frac{c_1}{\beta_1^2}\zeta_1\zeta_2 + \frac{a_1}{\beta_1^2}, \tag{28}$$

where we write ζ_i for the following:

$$\zeta_i = \int_0^1 \phi_i(x)dx = 2\Delta\bar{\xi}_i,\quad i = 1,2. \tag{29}$$

Eq. (29), taken into account Eq. (28), coincides with Eq. (24), i. e. the temperature (24), used in the first iteration, is exactly the average over the whole substrate area. If the difference between $\overline{T}_c$ and $\overline{T}_{c1}$ is slight, one iteration may be enough.

Numeric Calculations Results

The above formulae allow calculating the temperature distribution over the substrates of a single-stage and multistage TEC. In fact, to perform this calculation it is sufficient to be capable of finding the temperature distribution on the cold substrate of a single-stage TEC (see (16)), as evaluating the operational heat load on a TEC stage is a standard task of a TEC mathematical simulation.

Eq. (28) may be applied not only for a two-stage TEC, but also for a second stage of a multicascade TEC. For this purpose one has to know, even approximately, the temperature of the second stage hot substrate. Once the heat rejected by the previous cascades is found, it is possible to calculate the temperature distribution on any stage cold substrate of a multicascade TEC with the help of Eq. (28).

In practice it is often more important not to obtain the temperature two-dimensional field of the substrate but its average temperature $\overline{T}$ and the average temperature $\overline{T}_q$ of the contact area under the heat load. As an appropriate criterion of the distributional uniformity we take the difference $\Delta\overline{T} = \overline{T}_q - \overline{T}$. The analytical form of the heat conductance equation allows finding $\Delta\overline{T}$ easily. Thus we come to the following equation for a single-stage TEC:

$$\Delta\overline{T}_1 = \frac{Q_0 l}{\overline{S}_q K_f (L_1 L_2)(\alpha j + \kappa)} \left(\varphi_1 \varphi_2 - 4\Delta\overline{\xi}_1 \Delta\overline{\xi}_2\right) \tag{30}$$

and for a two-stage TEC:

$$\Delta\overline{T}_2 = \frac{c_1}{\beta_1^2} \left(\varphi_1 \varphi_2 - 4\Delta\overline{\xi}_1 \Delta\overline{\xi}_2\right). \tag{31}$$

As examples we give the results of $\Delta\overline{T}$ calculation for various kinds of heat sources localized on the cold side of the standard 127-couple 40x40 mm^2 TEC (1.15x1.4x1.4 mm^3 pellets) for different materials of the cold substrate (Table 1). The hot substrate temperature is taken 300 K. The reduced electric current is j=20 A/см. The heat source is placed in the centre of the cold substrate (ξ_1=20mm, ξ_2=20mm).

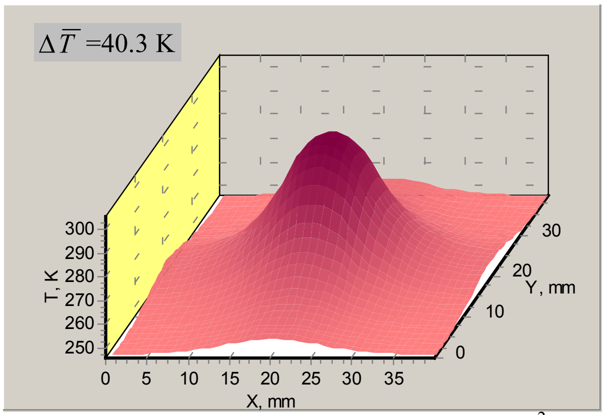
Table 1

The difference $\Delta\overline{T}$ of the temperature averaged over the area of the heat source and the temperature averaged over the whole substrate of the 127-couple 40x40 mm^2 TEC.

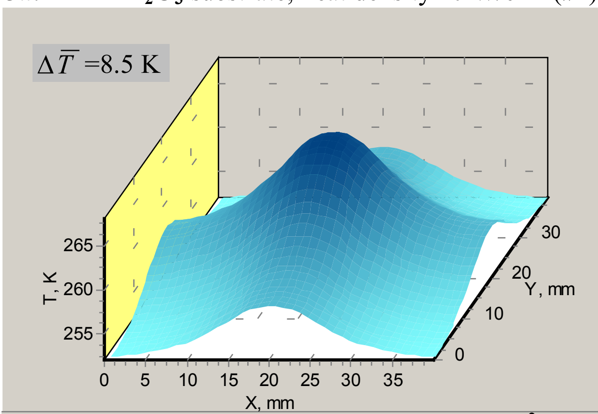
#	**Heat Load, W**	**Contact Area $2\Delta\xi_1$x$2\Delta\xi_2$, mm^2**	**Substrate Thickness, mm**	**Ceramics Material**	**Ceramics Thermal Conductivity, W/mK**	**Temperature Difference $\Delta\overline{T}$, K**
1	10	10x10	1	Al_2O_3	30	40.3
2	10	10x10	1	AlN	150	8.5
3	10	20x20	1	Al_2O_3	30	14.2
4	10	20x20	1	AlN	150	3.5
5	10	30x30	1	Al_2O_3	30	3.3
6	10	10x10	2	Cu	400	1.3

The data in Table 1 shows that the localized heat load with the heat density 10 W/cm^2 is poorly spread over the substrate of Al_2O_3. In this case even the AlN ceramics is not a way out. Only the 2mm thick copper substrate allows reducing heat losses to the extent of the calculation errors.

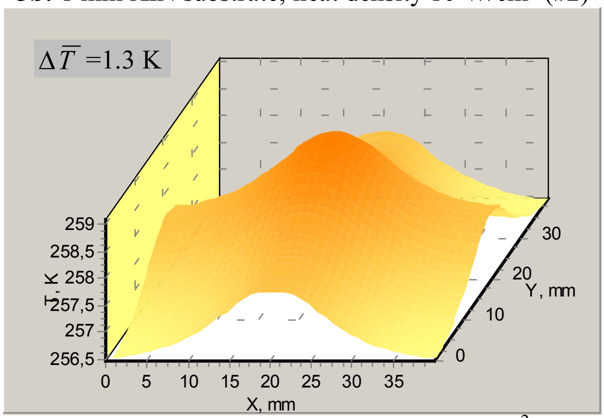
In Figure 3 we offer the illustrations of the temperature distribution fields for cases 1, 2, 6.



3a: 1 mm **Al_2O_3** substrate; heat density 10 W/cm^2 (#1)



3b: 1 mm **AlN** substrate; heat density 10 W/cm^2 (#2)



3c: 2 mm **Cu** substrate; heat density 10 W/cm^2 (#6)

Figure 3 Examples of 2D-temperature fields for cases 1, 2, 6 of Table 1

Another advantage of the method developed here is non-restriction to centre-symmetrical problems. Figure 4 yields some examples of 2D-temperature field for situation # 2 of Table 1 in case the rectangular heat source is shifted from the centre.

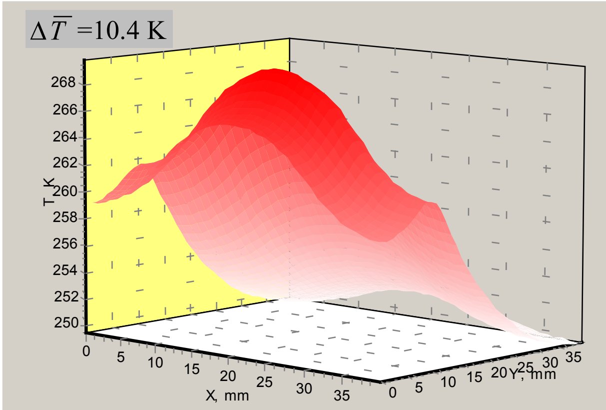


Figure 4 Example of 2D-temperature field for non-central localization of the non-square heat source: heat source 10 W, area $2\Delta\xi_1 x 2\Delta\xi_2$=20x5 mm^2. Centre ξ_1=20mm, ξ_2=10mm; substrate - case #2 of Table 1

Table 2 gives the data similar to those of Table 1 for two-stage TEC's based on Al_2O_3 0.5 mm thick ceramics.

Table 2

The difference $\Delta\overline{T}$ of the temperature averaged over the area of the first stage and the temperature averaged over intermediate substrate of two-cascade TEC's 1 and 2

Stage 1		Stage 2		Pellet Size, mm^3	Heat Load q_0, W	Current, A	T_h, K	$\Delta\overline{T}$, K
Cold Substrate, mm^3	Pellets Number	Hot Substrate, mm^3	Pellets Number					
4.8x4.8x0.5	20	6.4x6.4x0.5	58	0.4x0.4x1.5	0.2	0.3	300	0.69
"	"	"	"	"	0.3	0.5	"	1.28
"	"	"	"	"	0.5	"	"	1.55
"	"	"	"	"	0.6	0.4	"	1.45
8x8x0.5	16	10x10x0.5	48	0.7x0.7x1.15	0.6	0.8	"	1.12

Table 2 shows that correctly designed miniature TECs provide comparatively small heat losses on the intermediate substrate. Striking contrast of Table 1 and Table 2 data is due to the substrate sizes in the two-stage TECs considered. In case the sizes are large the thermal resistance between the pellet on the edge of the substrate and the heat load becomes too high, this pellet takes a lesser part of the heat load in comparison with the central pellets, which causes significant cooling losses.

Conclusions

In TEC design manufacturing a theoretical modeling should be of great reliability. Nowadays it is no problem to carry out *one-dimensional* computation and evaluate both specification and operation TEC parameters.

The paper proves that sometimes one-dimensional assessments are not enough. It becomes particularly evident when a TEC is required to perform either intense heat pumping or high temperature difference and in cases of a forced geometrical compromise between concentration and dispersion of the heat flow. In this cases the criterion $\Delta\overline{T} = \overline{T}_q - \overline{T}$ should be taken into account and detailed two-dimensional calculation check ought to be necessary.

Literature

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