

# Z-metering Development

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## Abstract

An easy-to-operate user-addressing device DX3065 for thermoelectric cooler (TEC) parameters measurement, as well as supporting mathematical and telemetry software are presented. It allows solving three problems posed by the Z-metering problem, its budding stem: electrical current little value for TEC-to-TEC control, thermal stationary transfer control and non-adiabatic dispersion verification via the current scanning and its partly compensation by the bipolar averaging. The device provides measuring TEC's AC resistance, time constant and figure-of-merit. The device can be applied as an express means for TEC certification and mass production quality management.

## Introduction

The Z-metering idea [1,2,3,4] implies the complex of requirements. The thermal ones are Joule heating and environmental effect insignificance. The temporal one is TEC non-stationary life extinction for the Z-metering approach is based on the thermal rate stationary equations [2]. The DX3065 allows meeting all the three due to the current scanning, mathematically grounded corrections and time telemetry read-out and analysis.

The DX3065 allows obtaining:

- alternating current (A.C.) resistance (R)
- TEC time constant of achieving the stationary state at the given measuring conditions
- figure-of-merit (Z) and derivation of the maximum temperature difference ( $\Delta T_{max}$ )

The DX3065 performs testing of various types of single-stage TE modules.

Additionally, it is possible to evaluate the  $Z$ -value of two-stage TEC. It is also possible to evaluate the time constant and quality of non-single stage TEC.

The paper is laid out as follows.

In Section 1 DX3065  $Z$ -measuring approach is surveyed. The mathematical grounding for bipolar averaging [4] is given. The electric current TEC to be tested is discussed. The theory-experiment correlation for bipolar testing and corrections involved in the approach is carried out.

Section 2 covers DX3065 TEC time constant application. The experimental results are discussed.

Section 3 embraces some essential technical aspects applying to A.C. resistance and the Seebeck voltage measurements. It also provides the glimpses of the device exterior and its software.

In the final section we conclude.

## **1. Z-metering: Bipolar Testing and Current Scanning**

In the  $Z$ -metering technique a TEC is examined in the actual arrangement. Some heat exchange with ambient vicinity takes place, the Joule heating is not of a zero value. So some correction factors [3] need to be taken into account to apply Harman's equation [2] to obtain the true Figure-of-Merit  $Z = \alpha^2 / kR = \alpha^2 / \kappa \rho$  [5] ( $\alpha$  - Seebeck coefficient;  $\kappa$  and  $\rho$  - thermal conductivity and electrical resistivity respectfully) and the corresponding  $\Delta T_{max}$ .

Here we offer a unified theoretical basis for meeting thermal requirements in testing TEC  $Z$ -value within the Harman approach.

Commonly the thermal rate equations for a 1-stage TEC are written the following way:

$$\begin{cases} \alpha T_0 - \frac{1}{2} I^2 R - k' \Delta T = a_1 (T_a - T_0) / N \\ \alpha T_1 + \frac{1}{2} I^2 R - k' \Delta T = a_2 (T_1 - T_a) / N \end{cases} \quad (1.1)$$

where  $N$  - pellets number,  $a_1$  - thermal conductance from the outer cold side,  $a_2$  - thermal conductance from the outer hot side ( $a_1 \neq a_2$ ),  $k'$  - effective pellets thermal conductance allowing for the air and electromagnetic field between them.

The term  $k'$  is described as:

$$k' = k(1 + b_{th}) , \quad (1.2)$$

where

$$b_{th} = B_{cond} + B_{rad} , \quad (1.3)$$

The  $B_{cond}$  and  $B_{rad}$  are corrections for inter-pellet thermal conductance values through air thermal conductivity and radiation, respectively:

$$B_{cond} = \frac{\kappa_{air}}{\kappa} \left( \frac{1}{\beta} - 1 \right) , \quad (1.4)$$

Here pellets filling term is:

$$\beta = \frac{Ns}{S} , \quad (1.5)$$

where  $S$  is cold side dimensions;

$$B_{rad} = \gamma \frac{S}{Nk} \sigma T_a^3 (1 - \beta) , \quad (1.6)$$

where  $\sigma$  - Boltzman constant,  $\gamma$  - thermal emissivity. As for (1.6) it can only be regarded as a rough estimate. We do not take into account air convection between the pellets as Grashof and Prandtl criteria show for this case [6].

In paper [8] equations (1.1) were solved and within the approximations:

$$\frac{a_1}{N} \ll k' , \quad \frac{a_2}{N} \ll k' \quad \text{and} \quad I \ll \frac{k'}{\alpha} , \quad (1.7)$$

and the following was found [7]:

$$\bar{T} = T_a \left( 1 + \frac{a_1 a_2}{N(a_1 + a_2)k'} \right) + \frac{I^2 RN}{a_1 + a_2} \left( 1 + \frac{a_1 + a_2}{4nk'} \right) + T_a \frac{\alpha I (a_2 - a_1)}{2k' (a_1 + a_2)}, \quad (1.8)$$

$$\Delta T = \frac{\alpha I}{k'} \left( T_a + \frac{I^2 RN}{a_1 + a_2} \right) + \frac{a_1 - a_2}{k' (a_1 + a_2)} \frac{I^2 R}{2} \quad (1.9)$$

We should devote great care to the requirements (1.7). These are environmental effect testing current constraints. The current limit (1.7) means the Peltier heats should be smaller than the vicinity effect. But along with it we see that (1.9) contains the value  $\frac{I^2 RN}{a_1 + a_2}$  -

Joule heating characteristics.

It is of an important concern that the Joule limitation for the current

$$I \ll \sqrt{\frac{(a_1 + a_2)T_a}{NR}} \sim \sqrt{(a_1 + a_2)T_a \frac{I_{\max}}{U_{\max}}} \quad (1.10)$$

is more meaningful than that in Eq. (1.7). It is illustrated by Fig. (1.1a) and (1.1b).

Judging by Fig. (1.1a,b) the small electric current criterion should be (1.10). Still the apparatus lower bound is not taken into account by any such kind of mathematics and its consideration is in store for us in this paper framework.

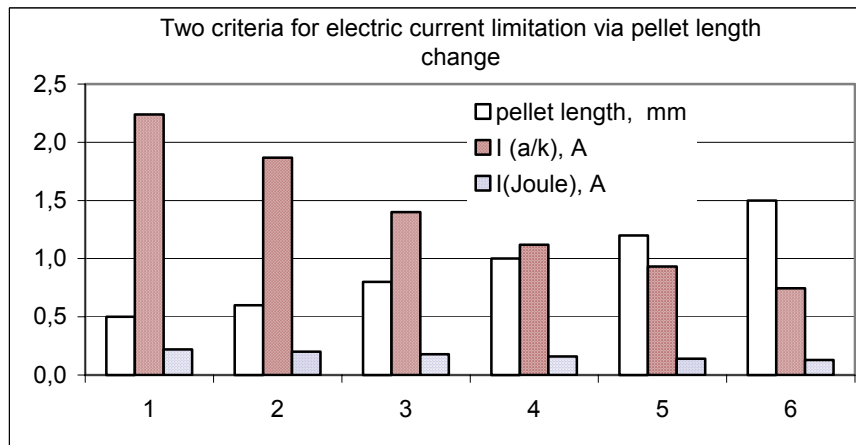


Fig.1.1a. The tested TECs are 1MC04-004-(05 – 15).

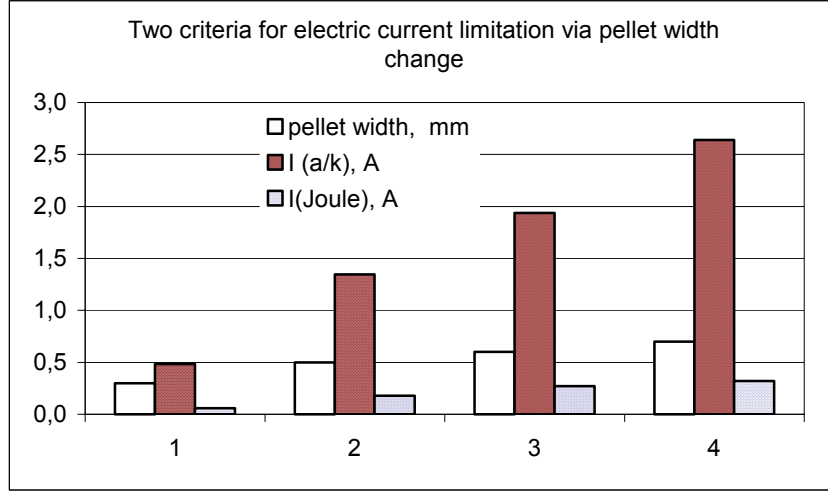


Fig.1.1b. The tested TECs are 1MT(03–07)-008-13

Allowing for (1.9) we obtain the following expressions for the thermoelectric power and voltage ratio:

$$U_{\alpha} = \frac{\alpha^2 IN}{k'} \left( T_a + \frac{I^2 RN}{a_1 + a_2} \right) + \frac{(a_1 - a_2) I^2 RN \alpha}{2k'(a_1 + a_2)} \quad (1.11)$$

$$\frac{U_{\alpha}}{U_R} = \frac{\alpha^2}{k'R} \left( T_a + \frac{I^2 RN}{a_1 + a_2} \right) + \frac{(a_1 - a_2) I \alpha}{2k'(a_1 + a_2)}, \quad (1.12)$$

or

$$\frac{U_{\alpha}}{U_R} = Z' \left( T_a + \frac{I^2 RN}{a_1 + a_2} \right) + \frac{(a_1 - a_2) I \alpha}{2k'(a_1 + a_2)}, \quad (1.13)$$

where  $Z' = \frac{\alpha^2}{k'R}$ .

So, the TEC value  $Z (= \alpha^2/kR)$  could be obtained as

$$Z = \frac{1}{T_a(1+b_T)} \left\{ \frac{U_{\alpha}}{U_R} (1+b_{th})(1+b_r) + b_A \right\}, \quad (1.14)$$

where

1)  $b_T = \frac{1}{T_a} \frac{I^2 RN}{a_1 + a_2}$  - Correction factor to ambient temperature

2)  $b_A = \frac{(a_2 - a_1)I\alpha}{(a_2 + a_1)2k}$  - Correction factor because of asymmetry of heat exchange with

environment;

3)  $b_{th} = B_{cond} + B_{rad}$  - Correction factor to pellet thermal conductivity due to additional heat flux from warm to cold side through the ambient (according to (1.4)-(1.6));

4)  $b_r = \frac{r}{NR}$  - Correction factor because of non-zero resistance  $r$  of TE module wires.

Due to the above-formulated correction factors equation (1.14) shows effect of actual arrangement of Z-metering technique.

The analyses of the correction factors share in the resulted true  $Z$  was previously done in papers [4,8] In the work [8] two problems were also considered: Z-measuring of a TEC mounted on the heat sink and of a two-stage TEC.

Of course, assumptions made in theoretical approach put many limitations in application of Z-metering technique for specific packages and arrangements of mounted TE modules.

Nevertheless there are two ways to use the technique for examination of assembled TE modules:

1. One can just do with the corrections outlined in comments on eq. (1.14). It should be kept in mind however that the uncertainty is inherent in all the corrections for the simple fact the input values are never known for certain. So, if possible, this way has to be thought twice about.

2. The advice on bipolar testing was given, for example, by Buist [4]. It allows excluding the asymmetry correction  $b_A$  (1.14). Let us analyze equations (1.12) - (1.14).

It is remarkable that the term  $b_A$  is a linear function of the current. Then marking one of polarities by (+), and the other by (-) we have:

$$\left(\frac{U_{\alpha}}{U_R}\right)_{+} = \frac{\alpha^2}{k'R} \left( T_a + \frac{I^2 RN}{a_1 + a_2} \right) + \frac{(a_1 - a_2)I\alpha}{2k'(a_1 + a_2)} \quad (1.15a)$$

$$\left(\frac{U_{\alpha}}{U_R}\right)_{-} = \frac{\alpha^2}{k'R} \left( T_a + \frac{I^2 RN}{a_1 + a_2} \right) + \frac{(a_2 - a_1)I\alpha}{2k'(a_1 + a_2)} \quad (1.15b)$$

Summing (1.15a) and (1.15b) we come to:

$$\left(\frac{U_{\alpha}}{U_R}\right)_{+} + \left(\frac{U_{\alpha}}{U_R}\right)_{-} = 2Z' \left( T_a + \frac{I^2 RN}{a_1 + a_2} \right)_a \quad (1.16)$$

So, the value Z could be obtained as:

$$Z = \frac{1}{T_a(1+b\mathbf{T})} \left\{ \left[ \frac{U_{\alpha}}{U_R} \right]_{averaged} (1+b_{th})(1+b_r) \right\} \quad (1.17)$$

In Fig. 1.2a,b we offer a common view resulted from bipolar testing and averaging

dependent on the electric current excluding  $(Z_{av} = \frac{I}{T_a} \left[ \frac{U_{\alpha}}{U_R} \right]_{averaged})$  and including

corrections.

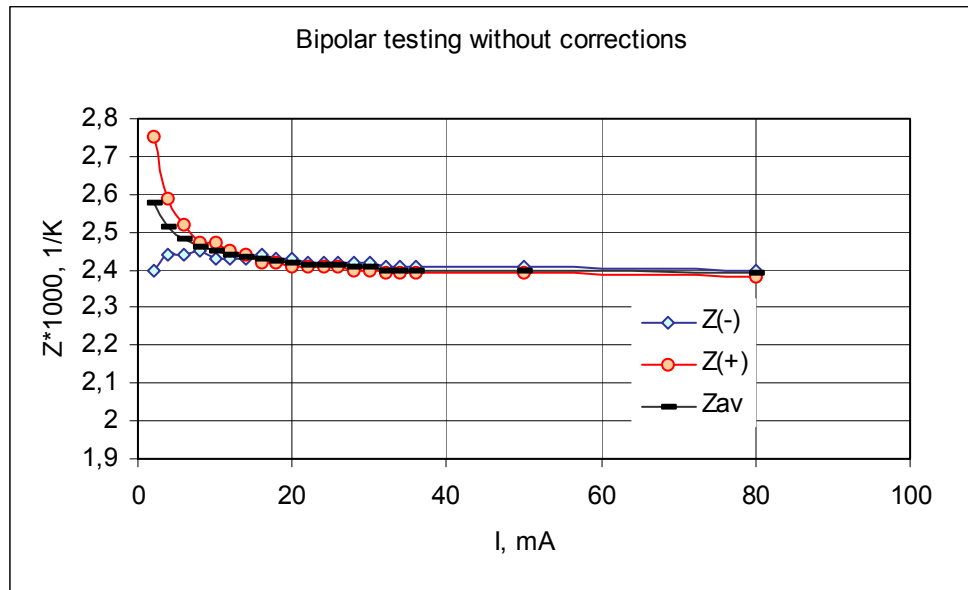


Fig.1.2a. The resulting figure-of-merit scanned by the electric current with no corrections considered (1MC06-032-15).

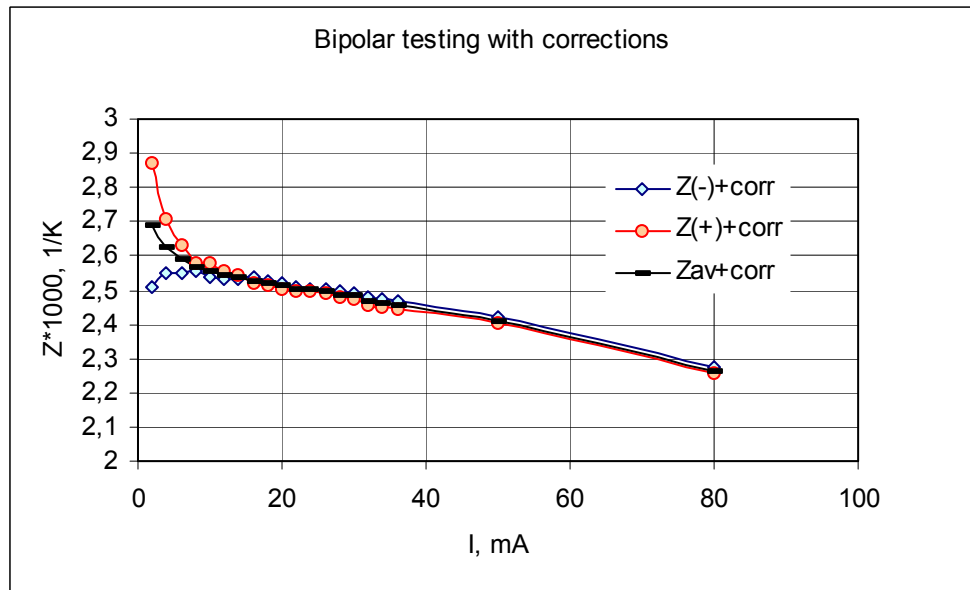


Fig.1.2b. The resulting figure-of-merit scanned by the electric current with corrections (1.14) considered (1MC06-032-15).

Armed with the electric current scanning technology we can also estimate the asymmetry corrections validity by the formula:

$$\left(\frac{U_{\alpha}}{U_R}\right)_{+} - \left(\frac{U_{\alpha}}{U_R}\right)_{-} = \frac{(a_2 - a_1)I\alpha}{k'(a_1 + a_2)} \quad (1.18)$$



In Fig. 1.3 we offer the comparison of the asymmetry correction calculated by the measured voltage ratios (left-hand side of eq. (1.18)) and by the theoretical expression (right-hand side of eq. (1.18)).

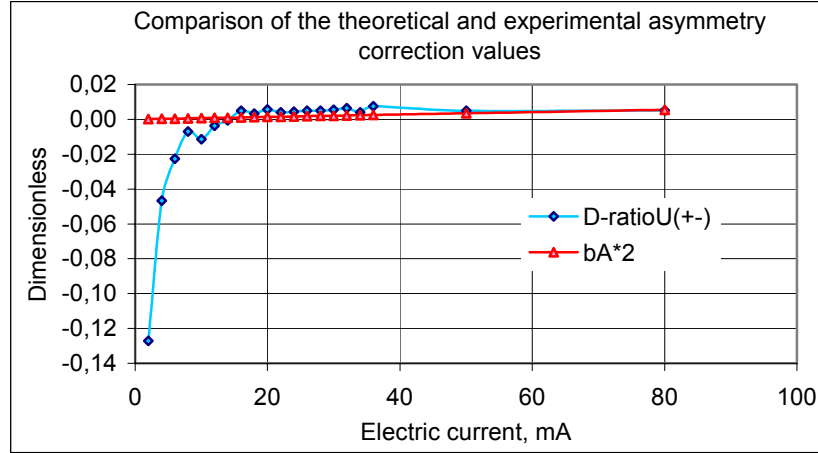


Fig. 1.3. The exemplary asymmetry correction fit (for the TEC 1MC06-032-15). The air convection thermal coefficient is  $4\text{W/m}^2\text{K}$ , cold and hot sides surfaces are  $64\text{mm}^2$  and  $80\text{mm}^2$  correspondingly, pellets thermal conductivity is  $1.35\text{W/mK}$ .

It is seen that the calculated asymmetry correction is only reliable for currents higher than some 10mA. Excluding it by the bipolar averaging seems an appropriate way out.

We believe the low current dissimulation can be explained by apparatus effect. The fact grounding it is that a variety of single-stage TECs was studied and the dissimulation proved to die out at the same current around 10mA. Therefore we suggest the initial current value for measurements not lower than 5mA.

It can be derived that the above theory is proved for the system TEC + heat sink in case

$a_0 / a_{\text{heat sink}} \ll I$ , which is actually always true for a real heat sink.

The general formulae for a two-stage module cold and hot sides are [8]:

$$\begin{cases} \alpha IT_0 - \frac{1}{2} I^2 R - k'_1 (T_1 - T_0) = \frac{a_1}{N_1} (T_a - T_0) \\ \alpha IT_2 + \frac{1}{2} I^2 R - k'_2 (T_2 - T_1) = \frac{a_2}{N_2} (T_2 - T_a) \end{cases}, \quad (1.19)$$

where  $T_0$ ,  $T_1$  and  $T_2$  - TE module's cold side, medium and hot side temperatures,  $a_1$  and  $a_2$  are the medium – cold and hot side thermal interjection coefficients respectively.

Z measurement for a two-stage case is admissible for the symmetrical requirements:

$$\frac{a_1}{N_1} = \frac{a_2}{N_2} = A = const, \beta_i = \frac{N_i s}{S_i} = const, k'_1 = k'_2 = k' \quad (1.20)$$

The bipolar testing also makes sense for a two-stage TEC but for accuracy reasons.

Unfortunately a mathematical transformation similar to that made for a one-stage TEC (see eq. (1.1) – (1.14)) yielding the immediate correlation between the TEC average temperature and the ambient temperature is not so simple and seems impossible so far. So for Z we obtain the following [8]:

$$Z\bar{T} = (1 + b_{th})(1 + b_r) \frac{U\alpha}{U_R} \quad (1.21)$$

Here  $\bar{T} = \frac{T_2 + T_0}{2}$  is the average module temperature set to equal the ambient temperature. The correction  $b_r$  satisfies (1.14).

$$b_{th} = B_{cond} + A_{conv} + (B_{rad} + A_{rad}) \quad (1.22)$$

The parameters  $B_{cond}, B_{conv}, B_{rad}$  are described above (1.4-1.6). Convection and radiation from external surfaces are correspondingly [8]:

$$A_{conv} = \frac{al}{\kappa\beta} \quad (1.23a)$$

$$A_{rad} = \frac{\gamma}{\kappa\beta} \sigma T_a^3 l \quad (1.23b)$$

In Fig. 1.4a,b we offer a common view resulted from bipolar testing and averaging dependent on the electric current excluding  $(Z = \frac{1}{T_a} \left[ \frac{U\alpha}{U_R} \right]_{averaged})$  and including corrections (1.21) – (1.23).

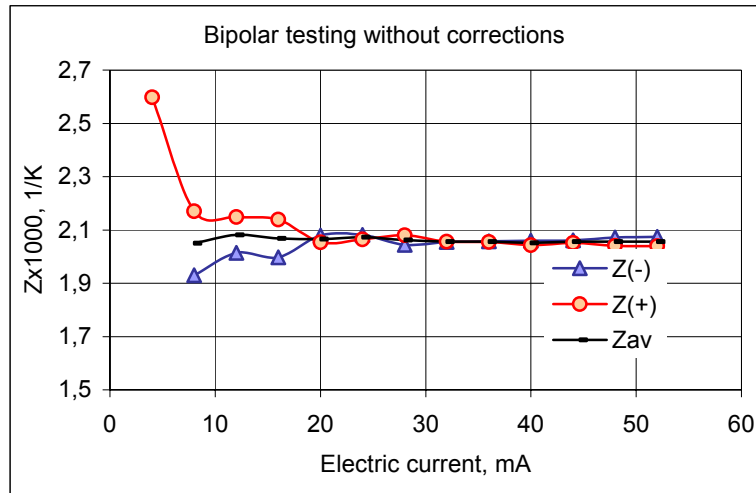


Fig.1.4a. The resulting figure-of-merit scanned by the electric current with no corrections for a two-stage TEC (2MC06-010-10).

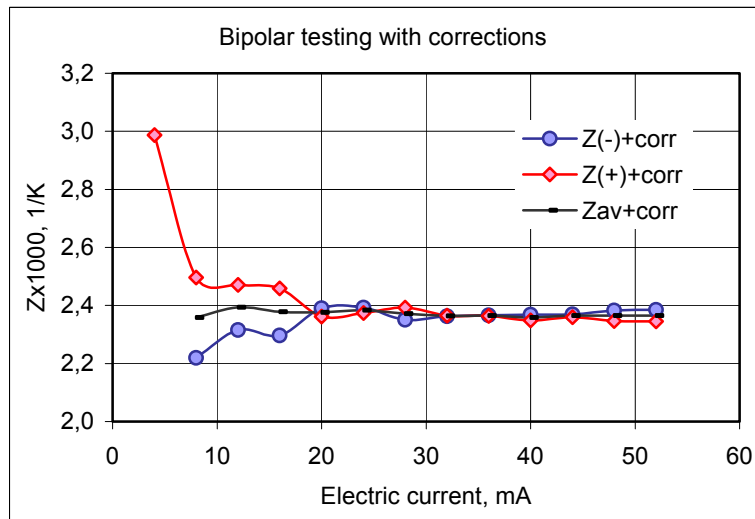


Fig.1.4b. The resulting figure-of-merit scanned by the electric current with corrections for a two-stage TEC (2MC06-010-10).

Now it is time to raise the long hanging up question: at what current value should we take the figure-of-merit as a final test result? The proper way out is the same for a one- and two-stage TEC.

We should take the lowest possible current at which the Z-values at both polarities do not undergo unphysical “tags” behavior. The Z-value at this current is the sought-for one.

For 1MC06-032-15 considered in Fig. 1.2a and 1.2b the currents lower than 5mA should be excluded. The figure-of-merit equals  $2.6 \cdot 10^{-3} K^{-1}$  (see Fig.1.2b).

For the two-stage 2MC06-010-10 it equals  $2.4 \cdot 10^{-3} K^{-1}$  (see Fig.1.4b).

## 2. Time Constant Testing

From the above it is clear that all the Z-metering mathematics is based on rate (balance) equations, which is only valid in case the time period considered is much longer than the time constant of the system. It should be proved by the DX3065 temporal initial sets.

A single-stage TEC thermal dynamics was studied previously by a number of authors [9,10]. It was shown that the differential equation result for the TEC transient dynamics can be presented as the following exponential superposition:

$$\Delta T(t, x) = \sum_{i=1}^{\infty} (A_n U_n(x)) e^{-m_n t} + \Delta T_{st}(x) \quad (2.1)$$

$\Delta T(t, x) = T - T_a$ ,  $T$  is the temperature of the TEC pellet point located at a time  $t$  and a generalized coordinate  $x$ ,  $U_n$  and  $m_n$  are the eigenfunctions and eigen-values,  $A_n$  are the thermal amplitudes,  $\Delta T_{st}(x)$  is the stationary result value.

The solution (2.1) analysis yields that the cooling process can be divided into two stages: irregular and regular [11]. The first one is dictated by the initial moment's conditions and is described by a multi-exponential interference. This stage fades out rather quickly, which is true in case TEC pellets thermal conductance is high enough, and the only exponent then characterizes the dynamics (regular mode). This phenomenon is applied in the DX3065.

Within the above-mentioned for estimations it is valuable to refer to the following qualitative ratio for the time constant  $\tau = 1/m_{\min}$  of single-stage TECs [3,12]:

$$\tau_0 \sim \frac{l^2 d c}{\kappa}, \quad (2.2)$$

where  $l$ ,  $d$ ,  $\kappa$ ,  $c$  are the pellet length, density, thermal conductivity and heat capacity. Qualitatively the following expression can be written for a one-stage TEC with heat capacity of the object to be cooled equal to  $C$ :

$$\tau \sim \frac{IC}{\kappa S N}, \quad (2.3)$$

Eq. (2.3) presents just ratio of the object heat capacity and total thermal conductance of pellets.

In the papers [10,12] it was also derived the current dependence of the time constant on the electric current exists. The DX3065 provides the chance to feel it.

The procedure of handling the time constant measurement data is as follows.

The temporal behavior of a single-stage TEC temperature difference is measured via the Seebeck voltage  $U_\alpha$  that is a corresponding proportional value:

$$U_\alpha = \alpha \Delta T \quad (2.4)$$

The measuring procedure is carried out both for two electric supply polarities. The data collection duration and time step can be varied. The obtained experimental data is then fitted by the following function:

$$U_\alpha(t) = U_{st\alpha} (1 - e^{-t/\tau}) \quad (2.5)$$

The exponential regression is based on the method of least squares. As its outcome the procedure provides the time constant  $\tau$  and the stationary Seebeck voltage  $U_{st\alpha}$ .

In Fig. 2.1 we offer the dynamics curve for 1MC06-032-15 at different currents.

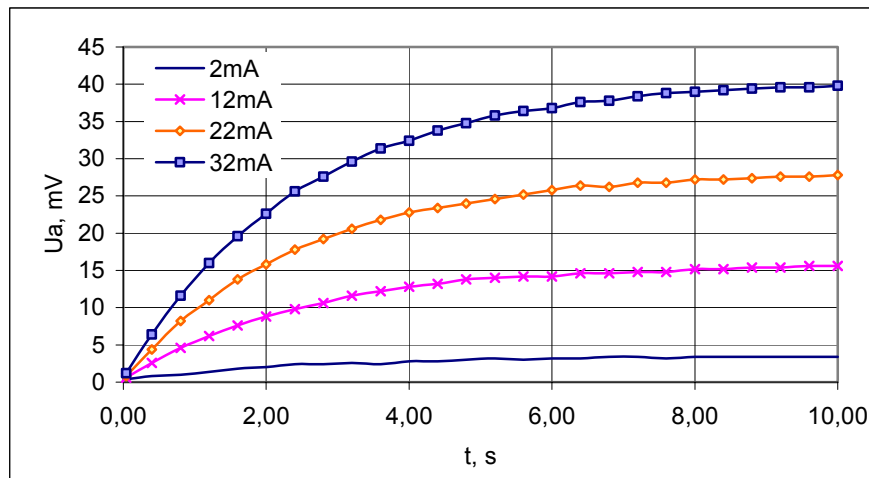


Fig.2.1. The Seebeck voltage vs time for different supply currents (1MC06-032-15).

In Fig. 2.2 one can see the current dependence of the time constants for the exemplary TEC above and the same series TEC with lesser pellet length.

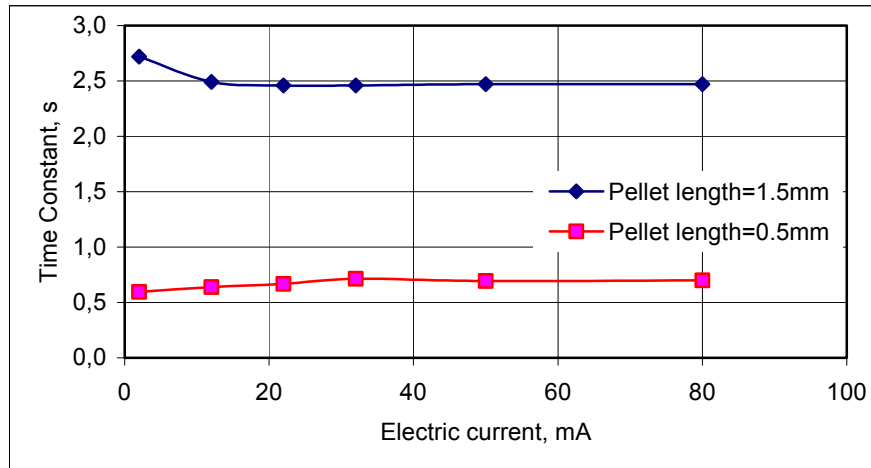


Fig.2.2. The Time constant vs current (1MC06-032-15 and 1MC06-032-05).

In the next figure non-stationary behavior of two TEC's varying by the pellets number is presented for the current 10mA. The difference in time constants is, apparently, due to different ceramics substrates heat capacity - pellets number ratio (eq. 2.2).

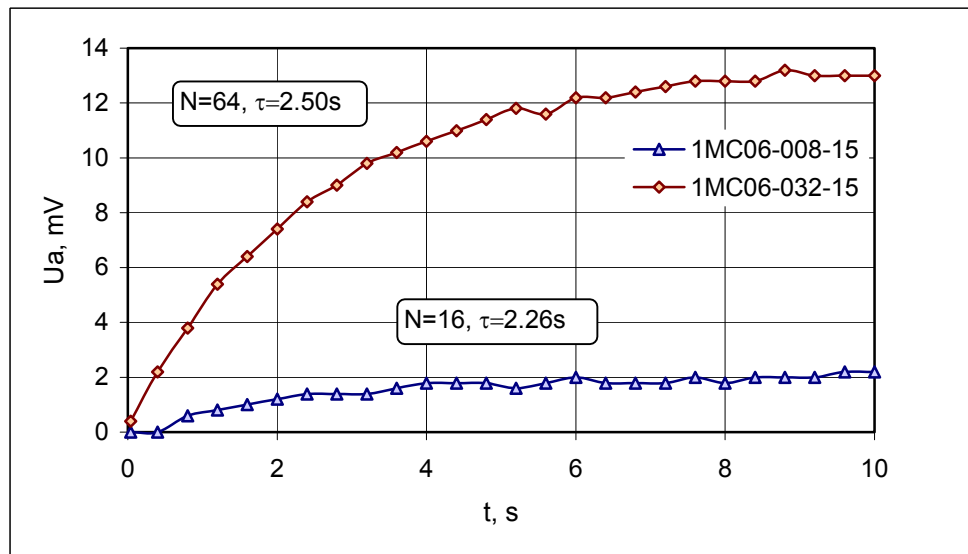


Fig.2.3. The time behaviour for two TEC's of different pellets numbers (the TEC tested are 1MC06-032-15 and 1MC06-032-15).

For a two- or more-stage TEC this simple ratio (2.3) is not applicable. However the time constant can be estimated by the temporal dependence of the Seebeck voltage and the

approach for obtaining the stationary voltage values is the same. In Fig. 2.4 the example of temporal behavior of the Seebeck voltage for a two-stage TEC is presented.

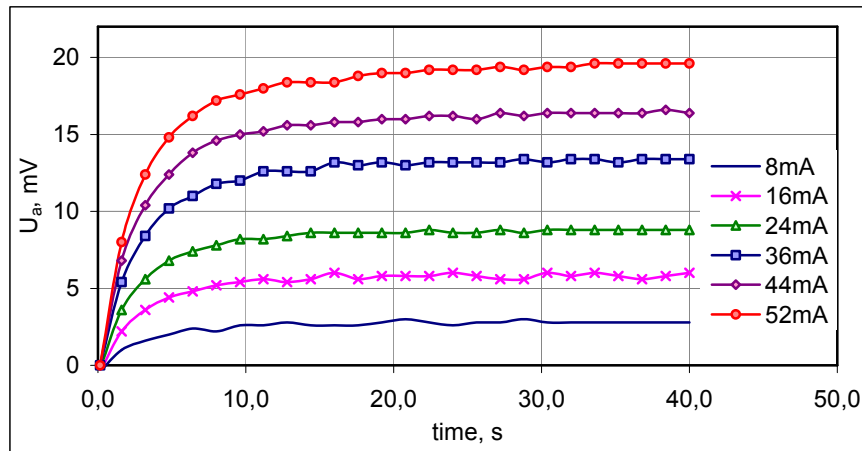


Fig.2.4. The Seebeck voltage vs time for different supply currents (2MC06-010-10).

### 3. Techniques

The DX3065 external view is presented in Fig. 3.1.

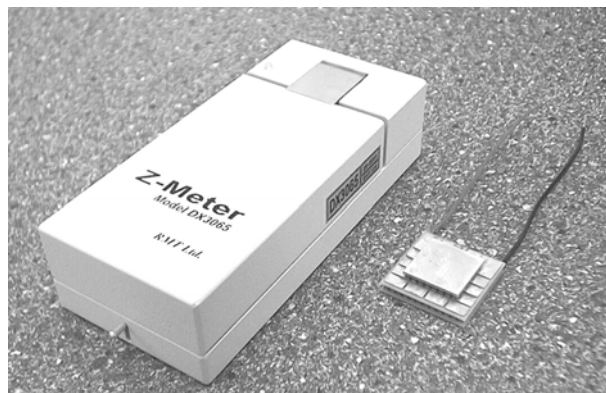


Fig.3.1. DX3065 en face.

The body of the DX3065 is made of an aluminum alloy. The metal body executes a function of a passive thermostat for measured TECs. Temperature of the body is measured with a digital thermometer, accuracy not worse than 0.1°.

A TEC to be measured is placed in this box.

The connection of the module is made through special connectors.

Simplified Functional Diagram of DX3065 is shown in Fig. 3.2.

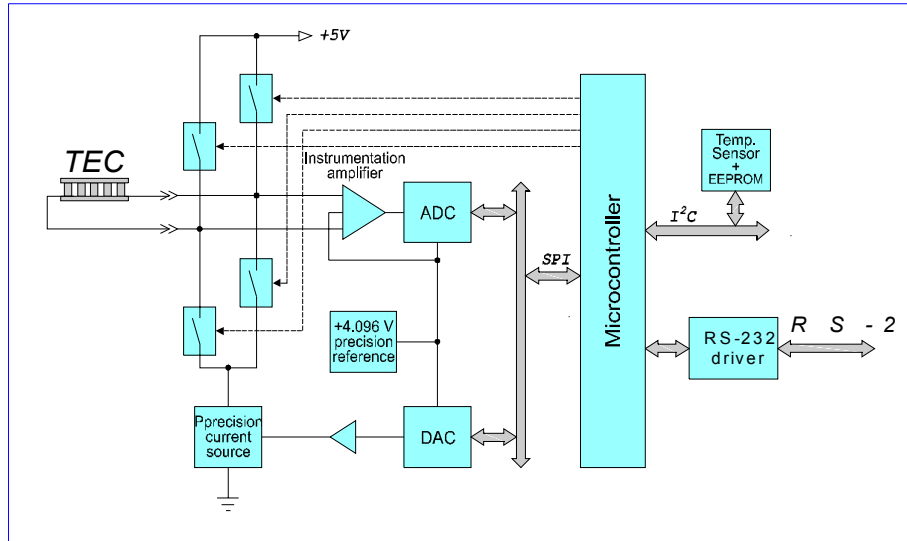


Fig.3.2. Functional Diagram of DX3065

The mechanisms of the two key parameters measurements (ACR and  $U\alpha$ ) are discussed here.

### 3.1 A.C. Resistance Measuring

Module is tested by AC current of small amplitude.

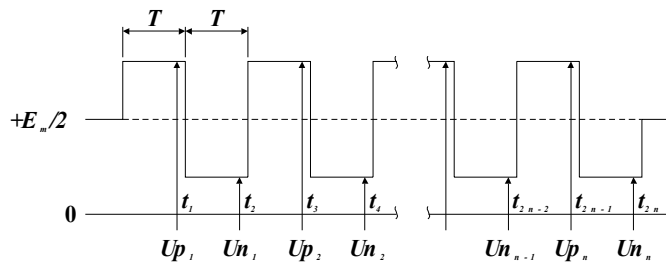


Fig. 3.3 Time diagram of AC  $R$  measurement

In the no input signal state the output voltage of the Instrumentation Amplifier is equal to  $E_m/2$  (Fig. 3.3). During AC resistance measuring the output voltage is sampled and measured by 12 bit ADC every time before  $I_m$  current reversing. The sampling points are marked as  $t_i$  in the figure. The voltage drops  $U_{pi}$  and  $U_{ni}$  corresponding to the positive and



negative polarities are used for a TE module resistance (R) calculation with the help of the following formula:

$$R = \frac{\sum_{i=1}^n (U_{pi} - U_{ni})}{2 \cdot I_m \cdot A_V \cdot n} \quad (3.1)$$

$A_V$  - voltage gain of instrumentation amplifier;  $n$  - total number of samples per measurement.

### 3.2 Seebeck Voltage Measurement

At measurement of  $U_\alpha$  parameters the small current is applied to a module periodically (with 50% duty circle).

Two successive measuring sessions are necessary to obtain the  $U$  and  $U_a$  values at different testing current polarities.

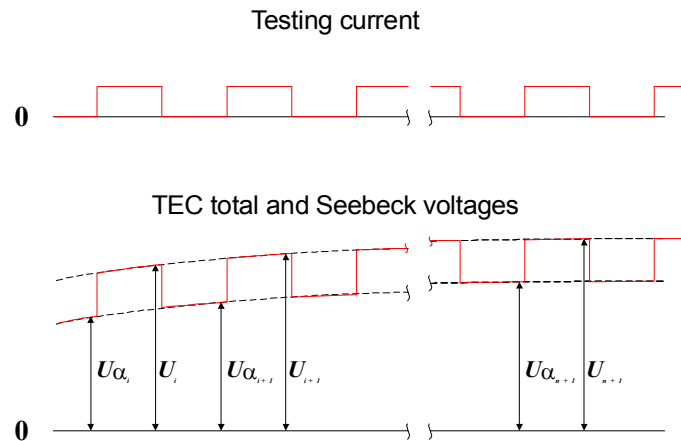


Fig. 3.4 Time diagram of the Seebeck voltage measurement

### 3.3 Software Glimpses

The operational window of the DX3065 supporting program is given in Fig. 3.5.

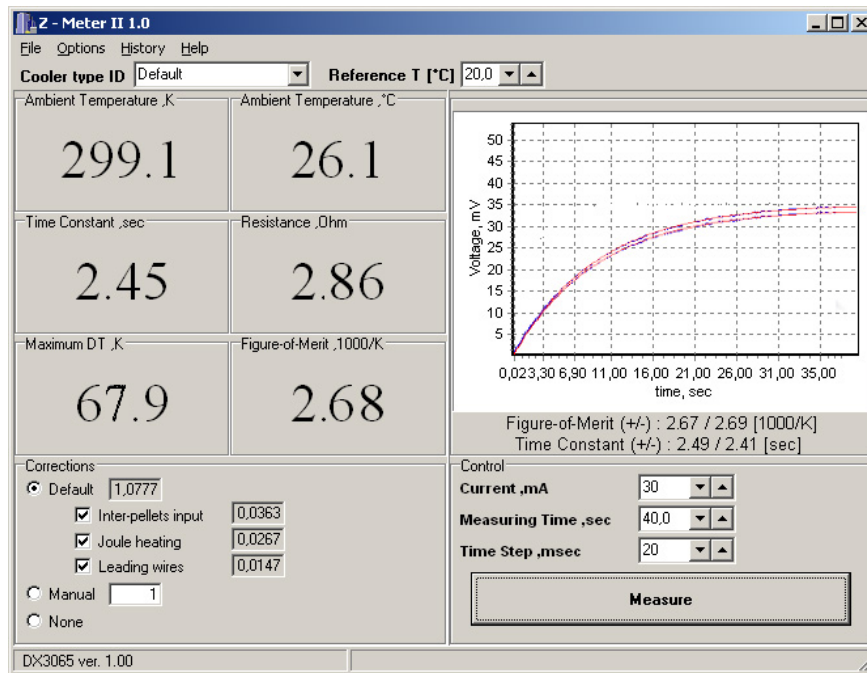


Fig.3.5. DX3065 software operational window.

#### 4. Conclusions

There were advised a new series of portable Z-meters suitable for examination of figure-of-merit, time constant and AC  $R$  parameters of TE coolers, which is very important for certification, quality control, failure monitoring and can be applied both in manufacturing and practicing.

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